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Synchronization Effects in a Submillimeter Josephson Self-Oscillator

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Abstract—**We present an experimental and theoretical study of injection-locking in an oscillator in the presence of noise. The experiment is performed with a Josephson point-contact self-oscillator heterodyne receiver irradiated by a very weak ≃ 1-THz signal. A general calculation of the oscillator response at low injection level is made based on the theoretical treatment of Stratonovitch. We show that the Josephson oscillator described by the RSJ model obeys the general locking equation in the presence of noise. We assume a simple evolution law of the oscillator spectrum as a function of detuning and calculate its response. The experimental results are compared with computer calculations and the implications are discussed.**

I. Introduction

We study here the partial synchronization of an oscillator in the presence of noise on a very weak external signal. We want more precisely to determine its spectrum as a function of the detuning relative to the injection frequency. The method of analysis does not depend on the type of oscillator. We present a theoretical and experimental study of synchronization in the superconducting Josephson self-oscillator mixer which is a system where noise effects are significant [1]. In this heterodyne mode of detection, a Josephson junction acts simultaneously as the local oscillator and as the nonlinear down-converter element. Our experimental interest is in applications of the device at submillimeter wavelengths with large frequency ranges of operation as may be required in a frequency-agile receiver. Therefore, the Josephson point-contact junction is coupled to a wide-band structure. In ths situation, noise plays a crucial role in the treatment of synchronization effects.

Injection locking in a Josephson point contact coupled to an X-band cavity was studied by Stancampiano and Shapiro [2] and Stancampiano [3]. In these experiments, noise was not considered since the junction was coupled to a high-Q resonator. The results could be interpreted on the basis of Adler's theory [4] of phase locked electronic oscillators because of the close similarity between the synchronization equations describing an electronic oscillator and the cavity-coupled Josephson oscillator.

For the wide-band Josephson self-oscillator mixer that we investigate here, Adler's theory cannot be applied since noise results in large natural oscillation linewidths [1]. We start from the general theoretical treatment of Stratonovitch [5] of injection in electronic oscillators in the presence of noise. This theory was used previously by Stephen [6] who calculated the effect of noise on the rounding of

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microwave-induced steps in resonant Josephson tunnel junctions. Here we determine the evolution of the fundamental component spectrum of the Josephson oscillator under very weak injection locking (no visible induced step).

In Section II we derive the Langevin equation of the phase in the Josephson oscillator relative to the small injected signal. Under this condition, the phase is in accordance with the general locking equation of an oscillator in the presence of noise. This is true for any nonresonating Josephson junction obeying the electrical RSJ model. The locking equation has analytical solutions for the linewidth narrowing at zero detuning and the mean frequency displacement at finite detuning. We deduce in Section III from these expressions an approximate law of evolution of the spectrum at finite detuning. Then we calculate with the model the response of the oscillator-mixer at the 4.75-GHz receiver IF frequency. Computer simulations are compared in Section IV to the experimental results with a Josephson heterodyne oscillator mixer receiver [7] irradiated with monochromatic 891-GHz laser radiation. The results allow a discussion of the limitations inherent to the oscillator-mixer mode of operation of a heterodyne self-oscillating Josephson receiver, and we evaluate the dynamic range of linear operation.

II. Modeling Injection Locking with Noise in the Josephson Self-Oscillator Mixer

A. The Basic Equations of the Josephson Device

A description of the device properties starts from the resistively shunted junction model of a Josephson point-contact junction with negligible capacitance (RSJ model) which gives the two equations corresponding to the circuit in Fig. 1

$$I_S + i_e(t) - i_p(t) - i_n(t) + i_f(t) = 0 \tag{1a}$$

$$d\phi(t)/dt = (2e/\hbar)v(t) = \omega(t) \tag{1b}$$

with

$$i_p(t) = I_c \sin\phi(t) \tag{1c}$$

where

- I_S bias dc current,
- $i_e(t)$ $= I_e(\cos\omega_e t + \phi_0)$ is the external monochromatic current
- $i_p(t)$ Josephson pair ac current of instantaneous angular frequency $\omega(t)$ and critical current I_c
- $i_n(t)$ dissipative normal current component
- $v(t)$ instantaneous voltage across the junction associated to the macroscopic quantum phase $\phi(t)$
- $i_f(t)$ fluctuation current defined by the autocorrelation function for a white noise spectrum

$$\langle i_f(t)\cdot i_f(t+\tau)\rangle = K\delta(\tau) \tag{1d}$$

- K noise intensity coefficient defining the physical process in the junction.

The submillimeter frequencies (~ 900 GHz) which we consider imply a junction dc bias smaller than the superconducting gap voltage, and we may assume a Josephson current amplitude I_c independent of the average voltage $V_0 = \langle v(t)\rangle$.

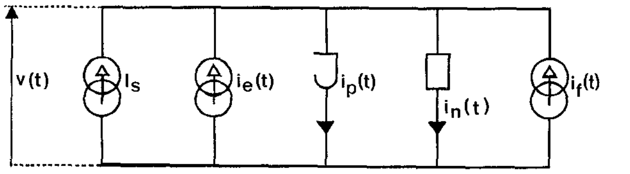

Fig. 1. RSJ model of the Josephson oscillator for the study of synchronization effects in the presence of noise.

A derivation of the Langevin equations associated with (1a)–(1d) allows the determination of the phase fluctuations across the junction. We proceed in two steps. We first write these equations without applied signals. Their solution gives the phase diffusion coefficient and the linewidth of the free Josephson oscillation. Next, we derive the new Langevin equation for a weak external monochromatic signal and obtain the equation of synchronization.

B. The Free Josephson Oscillator with Noise

We apply the method of slowly varying phases. The expressions relative to the dc and slowly varying quantities in (1a)–(1d) are

$$I_S - I_p[V(t)] - I_N[V(t)] + i_f(t) = 0 \tag{2a}$$

$$\dot\theta(t) = (2e/\hbar)v_f(t) \tag{2b}$$

where

$$V(t) = V_0 + v_f(t), \qquad V_0 = \langle V(t)\rangle. \tag{2c}$$

$\theta(t)$ represents the fluctuations of the Josephson phase associated with the noise voltage component $v_f(t)$. The mean frequency ω_0 of the Josephson oscillator is related to the dc bias voltage V_0 by the standard relation $\omega_0 = (2e/\hbar)V_0$.

Since $v_f(t) \ll V_0$, we expand $I_p[V(t)]$ and $I_N[V(t)]$ at the bias point in terms of $v_f(t)$

$$I_p[V(t)] = I_p(V_0) + v_f(t)/R_p(V_0) \tag{3a}$$

$$I_N[V(t)] = I_N(V_0) + v_f(t)/R_N(V_0) \tag{3b}$$

where $R_p(V_0)$ and $R_N(V_0)$ are the corresponding dynamic resistances.

Then the Langevin equation for the free Josephson oscillation becomes

$$\theta(t) - f(t) = 0 \tag{4a}$$

with

$$f(t) = R_D(2e/\hbar)i_f(t), \qquad R_D^{-1} = R_p^{-1} + R_N^{-1} \tag{4b}$$

and

$$\langle f(t)\cdot f(t+\tau)\rangle = k\,\delta(\tau), \qquad k = R_D^2(2e/\hbar)^2K. \tag{4c}$$

The associated Fokker–Planck equation relative to the probability distribution of the phase $P(\theta, t)$ is

$$\frac{\partial P(\theta,t)}{\partial t} = \frac{k}{2}\frac{\partial^2 P(\theta,t)}{\partial\theta^2}. \tag{5}$$

Equation (5) describes a Brownian-type motion with a diffusion of the phase given by

$$\langle[\phi(\tau)-\phi(0)]^2\rangle = \langle\theta(\tau)^2\rangle = 2D_\phi\tau \qquad (D_\phi = k/2). \tag{6}$$

The lineshape of the free Josephson oscillation is Lorentzian with a width

$$\delta\omega_0 = \langle\theta(\tau)^2\rangle/\tau = k. \tag{7}$$

C. The Synchronized Josephson Oscillator

A small external signal of frequency ω_e induces a phase modulation of the Josephson current (1c) which results from the coupled equations (1a) and (1b). The full calculation of the mixing process requires numerical computer calculation due to the current source modeling. In the small signal approximation we may represent this mixing process with its lowest order components of frequency $(\omega_e - \omega_0)$.

Then if $\omega_e \simeq \omega_0$ and $I_e \ll I_c$ (2a) relative to the dc and low frequency current components may be written

$$I_S - I_p[V(t)] - I_N[V(t)] - i_p(t) + i_f(t) = 0 \tag{8a}$$

where $i_p(t)$ is the mixing current component given by

$$i_p(t) = I_M(I_e)\sin(\phi - \phi_e). \tag{8b}$$

The phase difference

$$\theta(t) = \phi - \phi_e \tag{8c}$$

is now the instantaneous phase of the oscillator relative to the applied signal. The $I_M(I_e)$ dependence appears only in numerical calculations (see Section IV). At submillimeter wavelengths one has approximately $V_e = R_N I_e$, and the amplitude of the mixing component may be approximated with

$$I_M(Ve) \simeq I_c J_1(Ve/V_0) \tag{8d}$$

where $J_1(x)$ is the Bessel function of the first kind of order 1.

Proceeding as in Section II-B, the slowly varying voltage

$$v(t) = (\hbar/2e)\dot\theta(t) \tag{9}$$

is now referred to the dc voltage $V_{\text{step}} = (\hbar/2e)\omega_e$. V_{step} is the voltage at the first induced step which appears at larger power in the $I-V$ characteristic (full synchronization). Then (8a)–(8d) become

$$\Delta I - I_M \sin\theta(t) - (\hbar/2e)\dot\theta(t)/R_D + i_f(t) = 0 \tag{10a}$$

where

$$\Delta I = I_S - [I_N(V_{\text{st}}) + I_p(V_{\text{st}})]. \tag{10b}$$

The quantity $I_N(V_{\text{step}}) + I_p(V_{\text{step}})$ is the junction dc current corresponding to the bias voltage V_{step} and $R_D^{-1} = (R_p^{-1} + R_N^{-1})$ is the total dynamic junction resistance measured at $V_0 = V_{\text{step}}$ without external signal. Finally, (10) may be written

$$\dot\theta(t) = \Delta - \Delta_S \sin\theta(t) + f(t) \tag{11a}$$

with

$$\Delta = (2e/\hbar)R_D \Delta I = \omega_0 - \omega_e \tag{11b}$$

$$\Delta_S = (2e/\hbar)R_D I_M. \tag{11c}$$

This is the Langevin equation for the phase of the Josephson oscillation relative to the phase of the incoming signal at ω_e. The formulation is similar to the synchronization equation in the presence of noise for an electronic oscillator [8] or a phase locked loop [9]. In (11b), Δ represents the frequency difference between the Josephson free oscillation and the locking signal. It is proportional to the current ΔI in the junction and can be measured on the $I-V$ characteristic. Δ_S is defined as the synchronization frequency (in the absence of noise) which depends directly on the injected signal power. If $I_M(V_e)$ satisfies (8d), it may be easily shown that provided $V_e \ll V_0$

$$\Delta_S = \frac{\omega_0}{4\Omega^2}\sqrt{\frac{P_e}{P_0}} \tag{11d}$$

where $\Omega = \omega_e/\omega_c$ is the locking signal frequency in units of the Josephson characteristic frequency $\omega_c = (2e/\hbar)R_N I_c$, $P_e = V_e^2/2R_N$ is the external power coupled in the junction, and $P_0 = (1/8)\ R_N I_c^2$ is the Josephson oscillator power. The synchronization frequency Δ_s was defined in this form in the original paper by Adler [4] as well as in [2] and [3].

The resolution of (11) was studied by Stratonovitch [5] using the associated Fokker–Planck equation of the phase distribution function $P(\theta, t)$

$$\frac{\partial P(\theta,t)}{\partial t} = -\frac{\partial}{\partial\theta} \cdot [(\Delta - \Delta_S\sin\theta)P(\theta,t)] + \frac{k}{2}\frac{\partial^2}{\partial\theta^2}[P(\theta,t)]. \tag{12}$$

We have to calculate the spectrum of the synchronized Josephson oscillator which may be obtained by Fourier transform of the correlation function associated with the Josephson oscillation in (1)

$$k_p(\tau) = \langle i_p(t)\cdot i_p(t+\tau)\rangle \tag{13}$$

where $i_p(t) = I_c\sin\phi$ with $\phi = \phi_e + \theta(t)$. This requires a knowledge of $P(\theta, t)$, which must be calculated by integration of (12) under nonstationary conditions.

III. Calculation of the Synchronization Effects

The effects of synchronization on the spectrum of the Josephson oscillation in the presence of noise may be qualitatively described as a shift of the mean frequency relative to the unperturbed Josephson oscillation and an associated modification of the lineshape. There is no general rigorous solution of (12) available under nonstationary conditions. We have thus evaluated the effects of synchronization using specific solutions of (12) calculated by Stratonovitch [5].

A. Frequency Shift of the Mean Josephson Oscillation

This shift is determined in [10] by solving (12) for the stationary regime which yields

$$\langle\dot\theta\rangle = \langle\omega\rangle - \omega_e = \Delta\frac{\sin\pi D}{\pi D}[I_{\text{id}}(D_S)]^{-2} \tag{14}$$

where

$$D = 2\Delta/\delta\omega_0 \quad D_S = 2\Delta_S/\delta\omega \tag{15}$$

are parameters which measure the frequency shift and the synchronization frequency in units of the free oscillation linewidth $\delta\omega_0$. $I_{id}(D_S)$ is the Bessel function of imaginary argument and imaginary order. Equations (14)–(15) show that the frequency shift decreases when the noise level increases ($\delta\omega_0$ increases), or the applied signal level decreases (Δ_S decreases). The variation of θ as a function of the junction dc bias gives rise at higher power level to the rounding of the first induced step in the $I-V$ characteristic. It is this aspect of synchronization which has been treated theoretically by Stephen [6] and verified experimentally by Henkels and Webb [11].

B. Lineshape of the Josephson Oscillation

When there is no synchronization effect ($D_S \to 0$), the unperturbed Josephson oscillation lineshape is Lorentzian [5], [12] with a width $\delta\omega_0$. If there is synchronization and no detuning ($\Delta = 0$), the Josephson lineshape is still Lorentzian. The linewidth is obtained as in Section II-B from the diffusion coefficient of the relative phase $\theta(t)$. On the other side, (12) can be integrated under nonstationary conditions [4]. Then the width of the synchronized oscillation is given by

$$\delta\omega_S = \langle[\theta(\tau)-\theta(0)]^2\rangle/\tau = \delta\omega_0/I_0^2(D_S) \qquad (16)$$

where $I_0(D_S)$ is the zeroth-order modified Bessel function. If the level of the applied signal increases, V_e and D_S increase, $\delta\omega_S$ decreases, and the system reaches total synchronization. The dynamic resistance at the center of the Josephson step decreases following a similar evolution [6].

An approximate value of $P(\theta, t)$ has also been calculated [4] for a finite detuning ($\Delta \neq 0$), and a relatively large applied signal compared to noise ($\delta\omega_0 \ll \Delta_S$). However, a solution has not yet been given if the signal at ω_e is weak compared to noise ($\Delta_S \lesssim \delta\omega_0$), which is a situation typical of a Josephson self-oscillator receiver. In order to describe the evolution of the oscillation spectrum when detuning varies from large values (no synchronization, $D \gg 1$) to zero ($D = 0$), we assume that the lineshape remains approximately Lorentzian and that the linewidth $\delta\omega$ decreases linearly from $\delta\omega_0$ ($D \gg 1$) to $\delta\omega_S$ ($D = 0$) as a function of the displacement parameter $x = (\langle\omega\rangle - \omega_e)/\Delta$. The evolution of $\delta\omega$ with detuning Δ is given by

$$\delta\omega = \delta\omega_0(1-x) + \delta\omega_S x. \qquad (17)$$

This implies limited D_S values ($\lesssim 3$) so that the line is not distorted too much by the synchronization effects. The experimental results in Section IV will show that the approximation of (17) is reasonable.

A computer program determines the theoretical response of the oscillator-mixer as a function of the detuning Δ, with D_S as a parameter. It gives the power detected at the IF amplifier within the receiver bandwith, which we compare to our experimental results in the next section.

IV. Experimental Results and Discussion

The experimental study was performed at 891 GHz (HCN laser) with a wide-band receiver designed for submillimeter and far infrared waves. The experimental results were compared systematically to the computer calculations performed with the model of Sections II and III.

A description of the properties and performances of the receiver is given elsewhere [7]. We discuss briefly its characteristics. The Josephson junction is a Nb–Nb point contact operated at 4.2 K. It is shaped like a wide-band conical antenna for the far infrared signal. The laser beam is focused on the junction with an off-axis parabolic mirror adjustable from outside the cyrostat. The absolute value of the optical coupling coefficient of the mirror-conical antenna system is measured and ranges between 1 and 10 percent depending on the experimental conditions. The junction is in a coaxial matching structure for the intermediate frequency signals.

The IF signal is coupled to a cooled FET amplifier ($\omega_{if} = 4.75$ GHz, $B = 0.5$ GHz) through a cooled circulator (4.2 K). The noise temperature of the IF chain is 20 K or 150 K, depending on the amplifier used. The junction is isolated from the outside with a black polyethylene filter (4.2 K) and passive filters (4.2 K) on the dc bias leads. The whole structure is placed in an He exchange atmosphere at 4.2 K within a stainless steel shield. The experimental apparatus has an overall stability allowing measurements at periods of several hours.

The measurements which we perform to investigate synchronization effects deal with the mixing signal at the IF between the Josephson oscillation and the signal at ω_e which is directly related to the component $i_p(t)$ in (8b). In Sections II and III we have seen that injection locking manifests itself when the synchronization parameter D_S becomes of the order of unity, that is $\delta\omega_0 \gtrsim 2\Delta_S$ from (15). On the other side, these synchronization effects give visible effects on the IF frequency conversion signal only if $\Delta_S \gtrsim \omega_{if}$, which means $\delta\omega_0 \gtrsim 2\omega_{if} \simeq 9.5$ GHz.

A typical experimental result obtained under such conditions ($\delta\omega_0 = 23$ GHz) is represented in Fig. 2. It shows the evolution of the IF detected signal as a function of the junction current ΔI (see (10b)) for different values of the incoming FIR signal. The junction characteristics and the main experimental parameters are given in Table I. The total FIR coupling (optical losses and impedance mismatch) is here 2.10^{-2}. Each curve of Fig. 2 corresponds to a given input power expressed in decibels relative to its maximum $P_{in} = 120$ μW. There is no visible induced step in the $I-V$ curve within the resolution limit (a few tenths of a microampere) up to the maximum power. A characteristic feature of these curves is a progressive splitting of the detected signal. The depression between the two peaks and their separation increases with applied power. This separation may reach significant values, e.g., larger than the frequency difference between the two IF image frequencies (9.5 GHz). This behavior is related to the partial synchronization of the Josephson oscillation with the very weak monochromatic FIR signal. Fig. 3 represents for the same junction parameters the evolution of the IF power calculated with the model using D_S as a parameter. Equation (11b) gives the frequency detuning Δ as a function of the current ΔI (10b).

A comparison of the computer simulation of Fig. 3 with

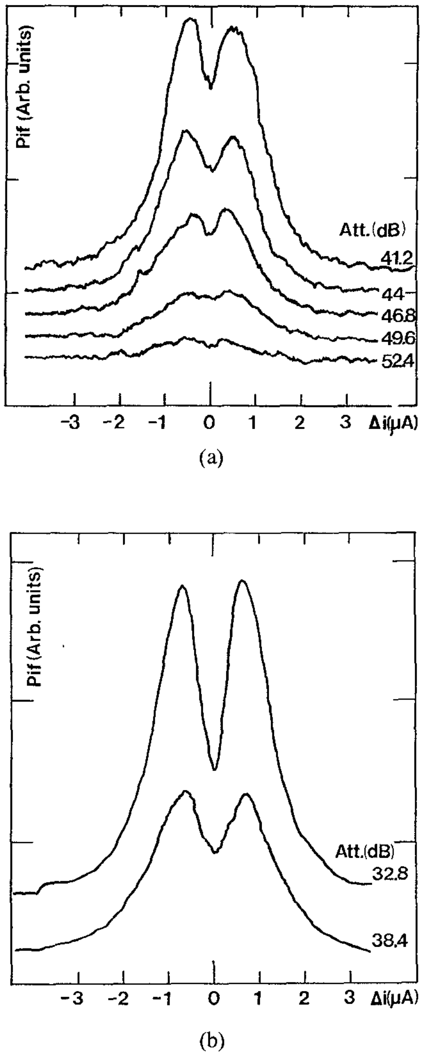

Fig. 2. (a) Intermediate frequency power P_{if} as a function of junction current ΔI (defined in (10b)) for different values of the submillimeter power at 891 GHz ($\omega_{IF} = 4.75$ GHz, $B = 500$ MHz). The latter is given in decibels relative to its maximum value $P_{in} = 120\ \mu$W at the input of the receiver. The junction parameters are given in Table I. (b) Same as Fig. 2(a) at higher applied power (IF gain reduced by four).

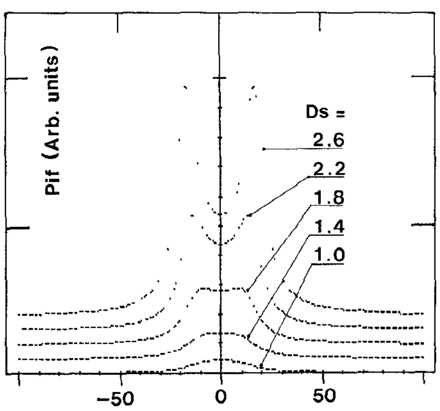

Fig. 3. Evolution of the IF power calculated with the model as a function of the detuning Δ for different values of the synchronization parameter D_S. Equation (11b) defines the relation between Δ and ΔI.

the experimental results in Fig. 2 results in the following observations:

a) The qualitative evolution of the IF detected signal is

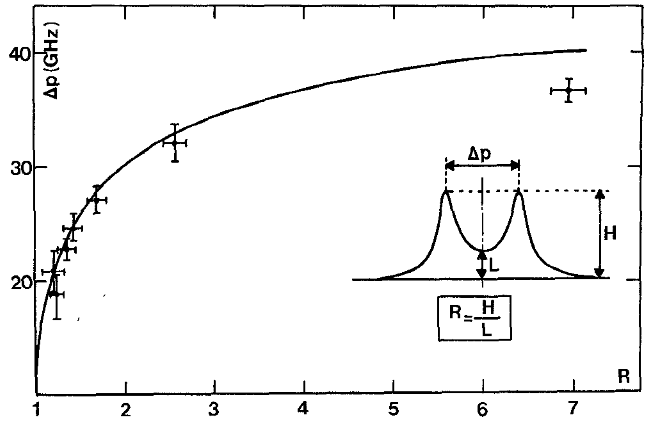

Fig. 4. Evolution of the parameters defining the shape of the IF signal $\Delta p = f(R)$ (see insert and text). The full line is the variation calculated from theory and the points correspond to the experiments reported in Fig 2.

TABLE I
EXPERIMENTAL PARAMETERS CORRESPONDING TO FIGS. 2 AND 4

R (Ω)	I_c (μA)	$\omega_c/2\pi$ (GHz)	$\delta\omega_0/2\pi$ (GHz)	I_S (μA)	P_{in} (μW)	Total F.I.R. coupling
39	36	680	23	64.5	120	$2\ 10^{-2}$

similar in both figures with the appearance of two peaks whose separation increases with applied power.

b) The shape of the curves may be characterized approximately by the functional dependence of the splitting Δp between the two peaks on the relative depth R of the depression (power at peak/power at minimum where $\Delta = 0$). We show in Fig. 4 the evolution of $\Delta_p = f(R)$ deduced from Fig. 2 (experiments) and Fig. 3 (computer simulation). It must be emphasized that there is no fitting parameter in the calculations leading to Fig. 4. The good agreement between measurements and calculations in Fig. 4 indicates that the model gives a proper account of the synchronization effects.

c) The experimentally observed evolution of the IF signal level P_{if} with applied power is slower than predicted by (11d) which is used to obtain experimental values for $D_S = 2\Delta_S/\delta\omega_0$. However, the orders of magnitude are well verified mainly at low power level ($D_S \lesssim 2$). This indicates that the power level effectively used in the analysis of the locking phenomenon is not taken into account well enough by (11d). The disagreement related to the calculated power which fits the observed synchronizations effects may have two possible explanations. 1) We have assumed an HF junction impedance equal to the normal resistance R in the calculation of the dependence of the power effectively coupled with D_S. However, this is not absolutely true if the Josephson frequency ω_0 is near ω_e [13], [14]. 2) We do not take into account the junction coupling with the IF circuit in (10)–(11). The latter may be represented by an impedance Z_{if} in parallel with R_D (40 Ω). Then the IF circuit would be matched to 10 Ω in the 4.5–5-GHz band. Outside this bandwidth and within the 0–10-GHz range, Z_{if} is not known, although its value may be situated in the 10–50-Ω scale. The net result of this impedance ought to be a

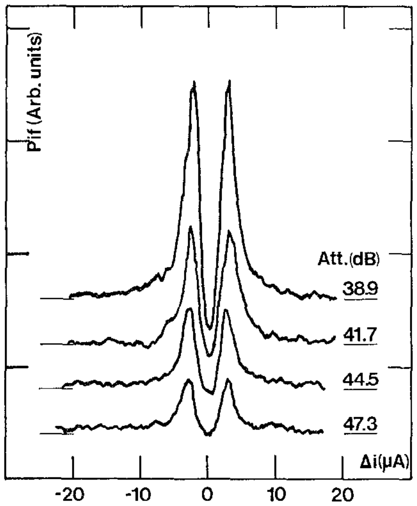

Fig. 5. Intermediate frequency power P_{if} as a function of junction current ΔI (see Fig. 2) when the Josephson linewidth $\delta\omega_0$ is relatively narrow ($\simeq 3$ GHz).

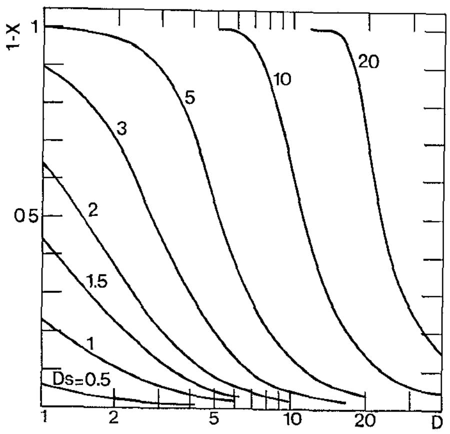

Fig. 6. Conditions of partial synchronization effects: relative frequency displacement $1-x=(\omega_0-\langle\omega\rangle)/(\omega_0-\omega_e)$ as a function of the normalized detuning D with $D_S=2\Delta/\delta\omega_0$ as a parameter.

reduction of R_D, Δ_S (see (11c), D_S, and of the calculated effects of synchronization.

We have discussed up to now situations where the Josephson natural linewidth has the same order of magnitude as the IF ($\delta\omega_0=23$ GHz, $2\omega_{if}=9.5$ GHz) and where synchronization gives rise to strong distortions of the observed signals. We have also performed experiments with junctions presenting much broader oscillation linewidths (up to ~ 100 GHz). The general features are similar and a depression appears in the junction response. However, the assumptions underlying the calculations for the junctions of narrower linewidth do not apply. Now, for synchronization conditions defined by a given value of D_S, a detection of the Josephson oscillation with large $\delta\omega_0$ requires a rather large applied power at ω_e. As an example with $D_S=3$ and $\delta\omega_0=100$ GHz, (11d) gives $P_e/P_0\simeq 0.4$, and calculations using the results of Section II-C are no longer valid.

On the other hand, we have investigated the case of relatively narrow Josephson linewidths ($\delta\omega_0=3$ GHz, $\delta\omega_0/\omega_0\simeq 3.3\ 10^{-3}$) and show the results in Fig. 5. The general features here are very similar to those of standard electronic oscillators. Here the two lines correspond to the two image frequencies of the heterodyne detection, and their separation is 9.5 GHz. Their linewidth is the Josephson linewidth given by (7). Under these conditions, injection does not produce significant effects on the mixing signal. The latter result is well confirmed by the calculations.

V. Conclusions

In conclusion, we have investigated a theoretical model of injection locking of an oscillator in the presence of noise. Our experimental study of the Josephson oscillation shows that the model gives a fair quantitative account of the effects of partial synchronization on a weak monochromatic signal. In particular, the evolution of the detected IF signal which translates the modification of the Josephson oscillation is described satisfactorily (Fig. 4). A practical consequence of this modification is a reduction of the dynamic. This effect is important when the free Josephson linewidth $\delta\omega_0 \gtrsim \omega_{if}$. Then we find that the effect of saturation begins if $D_S \gtrsim 2$. As an example, in the experimental case corresponding to Table I and Fig. 2 ($\delta\omega_0=23$ GHz and $\omega_{if}=4.75$ GHz), the maximum signal power at 1 THz before saturation is $2.5\ 10^{-3}P_0\simeq 1.6\ 10^{-11}$ W. On the other hand, minimum detectable power in the system is $1.3\cdot 10^{-13}$ W when the IF chain with 150-K noise temperature is used, and all sources of conversion loss are included. Then the corresponding calculated dynamic range is $\simeq 20$ dBs which compares well with the 17-dBs value measured in the experiment.

The effects of synchronization in a Josephson self-oscillator receiver can be decreased by reducing the natural oscillation-linewidth with an adequate shunt. This can be seen from the expression $D_S=2\Delta_S/\delta\omega_0$ which shows that for a fixed D_S, the synchronization frequency Δ_S decreases with $\delta\omega_0$. Such a solution will be possible with the development of Josephson tunnel microjunctions (edge junction) that include a shunt tailored to affect only the low frequency noise components.

Generally speaking, our approach may be used to describe the effects of partial synchronization in any type of oscillator (i.e., electronic, laser), provided the diffusion equation of the phase (e.g., (11a)) is derived and the statistical properties of the noise sources (1d) are known. Then a calculation of the system response as a function of the normalized detuning $D=2\Delta/\delta\omega_0$ requires only a knowledge of the locking parameter $D_S=2\Delta_S/\delta\omega_0$. Our experiments show that when the external signal is very weak, the spectrum of the partly synchronized oscillator can be completely defined by $x=(\langle\omega\rangle-\omega_e)/\Delta$ with $\Delta=\omega_0-\omega_e$. We have plotted in Fig. 6 the relative displacement $1-x=(\omega_0-\langle\omega\rangle)/(\omega_0-\omega_e)$ as a function of the normalized detuning D. These curves enable us to predict the conditions of partial synchronization.

As an example, a relative displacement $1-x\simeq 10$ percent is realized if $D\gtrsim 2D_S$, i.e., $\Delta \gtrsim 2\ \Delta_S$. The conditions of full synchronization can also be deduced from the curves. An extension of the study to larger injected signals requires a more accurate determination of the oscillator spectrum which is a difficult problem yet to be solved.

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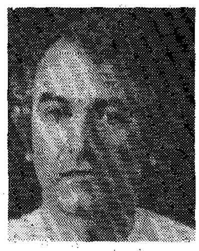

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A High-Power *W*-Band (90–99 GHz) Solid-State Transmitter for High Duty Cycles and Wide Bandwidth

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Abstract —**A high average power *W*-band solid-state transmitter using a 2-diode and a 4-diode IMPATT power combiner has achieved over 1.89 W and exceedingly versatile performance over a broad range of pulsewidths and duty cycles with a tunable bandwidth from 90 GHz to 99 GHz.**

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I. Introduction

A NEW GENERATION of millimeter-wave systems will demand high-power solid-state *W*-band transmitters [1]. Millimeter-wave tracking radars and active seekers for precision guided munitions need small, lightweight, reliable solid-state transmitters capable of operating over a broad range of pulsewidths, duty cycles, and

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